Book of Numbers: Exploring Jewish Mathematics and Culture at a Jewish High School

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Background notes: The word Torah refers narrowly to the books of Genesis, Exodus, Leviticus, Numbers and Deuteronomy, but can also include the entire Hebrew bible (the body of scripture non-Jews call the Old Testament and Jews call the Tanakh or Written Torah). In this article, the reader will see the usual Talmudic citation convention where "Ketubot 93a" refers to a particular side (in this case, front) of a particular two-sided leaf (93) from a particular tractate (Ketubot). Talmud refers to Jewish rabbinical oral tradition, which was written down over 1300 years ago. The designations BCE (Before the Common Era) and CE have been in use for several decades as more neutral substitutes for BC and AD, respectively, to keep the same dating system while being more respectful of non-Christian faiths. Hebrew words are written right to left.

Introduction

The domain of culturally relevant mathematics includes the recognition that mathematics has been present in every culture, the mathematical achievements of cultures, the effect of mathematics on any culture, and the right for all people to acquire mathematical power (Hatfield, Edwards, Bitter, et al. 2000). While aware of some literature on culturally relevant mathematics (Gutstein, Lipman, Hernandez, et al., 1997) and supportive organizations (Benjamin Banneker Mathematics Association, www.bannekermath.org; TODOS: Mathematics for ALL, www.todos-math.org), I had not had the encouragement and opportunity to integrate culturally relevant mathematics into my teaching in a comprehensive way. The opportunity came recently when I took a leave of absence from a university mathematics education position to upgrade my experiential base with full-time precollegiate teaching experience. For the next two years, I worked as mathematics department chair and teacher at a pluralistic Jewish community high school in a large city in the southern United States. Most students were affiliated with Reform or

Conservative congregations, though a few students had Modern Orthodox affiliation and a few were non-Jews. In the spirit of NCTM (2005) that "highly qualified teachers of mathematics not only understand – but also invest in – the particular culture of their students and school", I sought, compiled, created, and implemented connections to Jewish culture with a range of students (their standardized test scores went from the 40^{th} to 99^{th} percentile) in a range of courses (algebra, geometry, precalculus, calculus) as well as in school assembly presentations. To support this endeavor, I supplemented my own Judaics knowledge and got more acquainted with the school's approach to Judaics by attending daily assembly presentations and sitting in on an 11^{th} grade Judaics course during my planning period.

Significant material on Jewish mathematics appears in a few mathematics history textbooks (Katz 1993) and in moderately advanced Judaics books (Littman 1989, Gabai 2002), but there does not appear to be a single book or bibliography specifically and comprehensively on Jewish mathematics from which a classroom teacher could readily teach. This article shares and reflects upon a broad cultural overview of examples from many sources I shared with my high school students (and, occasionally, with general adult audiences at lectures). Clearly, I used only a subset of these examples in any one course I taught, and a few examples I did not get around to using in any, but am including them to offer a more comprehensive collection for other instructors.

The examples are intended to be of interest to Jew and non-Jew because: (1) most examples have (or may point to) counterparts in other cultures; (2) it is possible and profound to help students experience their culture as something dynamic and interdisciplinary with a nurturing egalitarian worldview that places "their history within a universal context where being part of an ethnic group is a reflection – not a separation – of their humanity" (Aceves 2004, p.

275); (3) in many countries, a large fraction of students belong to a faith tradition (e.g., Christianity, Islam) whose origin includes significant Jewish context; also, there has been increasing mainstream interest in various aspects of Jewish culture, due to media coverage of the Middle East and involvements in Kabbalah (Jewish mysticism) by celebrities such as Madonna, Demi Moore, Rosanne Barr, Sandra Bernhard, etc.

While Judaism has been viewed as many things (e.g., culture, race, religion, civilization, nationality), this article's focus is on culture for several reasons. As Lemish (1981) states: "All attempts to categorize or identify Jews as an ethnic, religious, or national group are simply inadequate and incomplete… Perhaps the closest any identification can come is to view the Jews holistically as a culture" (p. 28). Indeed, students at the Jewish high school generally viewed and described their own identity that way (e.g., "Jewishly-identified, but not real religious") and the school acknowledged this by emphasizing culture more than religion at assemblies, working in small doses of religion gradually in a participatory, non-coercive manner. So when we reference Jewish religious ideas, it should be kept in mind that many students (and adults) refer to or follow certain Jewish religious practices more as a matter of cultural solidarity and familiarity than as an active explicit theological belief.

However, there has been much exclusion (by Jews as well as by non-Jews) of Jews from multiculturalism, the extent of which and various explanations for are reviewed by Langman (1999). One of my goals is to help articulate and explore broader, more visible roles for Jewish culture in multicultural mathematics. While Jews are not underrepresented in careers requiring significant mathematics, it is a loss for all that Jewish culture is underrepresented in the area of multicultural mathematics. A popular commercially available set of 16 multicultural classroom posters includes posters such as "Math of Egypt", "Math of Babylon", and "Math of Arabia", but not “Mathematics of Israel”. The NSF-funded Ethnomathematics Digital Library (2006) has entries for 132 cultural group categories, but “Jewish” is not one of the categories (though “Hindu”, “Islamic” and “Muslim” are, for example).

Teaching Jewish mathematics as history and culture also clearly follows the separation of church and state that would support implementation in public schools as well. Thus, we are not “teaching religion”, even if doing some teaching *about* religion-- in the balanced spirit of the broadly-endorsed school textbook by Schippe & Stetson (2005).

Using Culture for General Motivation: Quotations and ‘Firsts’

One of the simplest and most immediate ways I found to begin integrating culturally relevant mathematics was to add to the physical environment of the classroom. To motivate students who personally took their Judaism much more seriously than their mathematics, I prominently posted on the walls of my classroom some of the explicit support within Jewish tradition for learning mathematics. Here are two examples:

“[One] who wishes to attain human perfection should study Logic first, next Mathematics, then Physics, and lastly Metaphysics.” -- 12th-century rabbi, physician, philosopher Moses Maimonides, in his *Guide for the Perplexed*

“The statement of Galileo that ‘the great book which ever lies before our eyes—I mean the Universe—is written in mathematical language and the characters are triangles, circles, and other geometrical figures’ applies as well to the Halakhah [Jewish Law]. And not for naught did the Gaon of Vilna [a very influential 18th century rabbi who also authored mathematics books] tell the translator of Euclid’s geometry into Hebrew [R. Barukh of Shklov], that ‘To the degree that a man is lacking in the wisdom of mathematics he will lack one hundredfold in the wisdom of the Torah.’” – Rabbi Joseph Soloveitchik, one of the 20th century’s most influential scholars in politically-conscious Modern Orthodox Judaism, in his *Halakhic Man*

Mathematical “firsts” have come from many cultures, and can be a source of inspiration for students from that culture. Because the Jewish written record is unusually old, and because it has received a great deal of scholarly attention, it has been credited with a large variety of mathematical firsts. For example, Wainer (1996) shows a representation of possibly the first statistical graphic (made around 1400 BCE and uncovered from the Qumran Caves) -- a bar chart of 2 population counts 38 years apart for each of the 13 tribes.

The Talmud (Ketubot 93a) contains the first "fair division problem" in recorded history: how to divide an estate between three creditors owed 100, 200, and 300 units. An estate of 100 is divided using equal division so that each creditor gets $33\frac{1}{3}$ units. An estate of 300 is divided using proportional division, so that each creditor gets half of what he is owed: 50, 100, 150. For an estate of 200, however, the rationale for the Talmud's allocation (50, 75, 75) seems mysterious. These 3 cases were not reconciled mathematically until 1984 (Aumann & Maschler 1985; Hill 2000) as the "nucleolus solution" minimizing the largest dissatisfaction among all subsets of creditors.

Jewish mathematician Levi ben Gerson's 1431 work *Maasei Hoshev* introduces the technique of mathematical induction "somewhat more explicitly than his Islamic predecessors" (Katz 1993, p. 279). Using this technique, he proved that the number of permutations of n elements is the product of the first n natural numbers. (More about this mathematician appears in Simonson (2000).) Interestingly, this particular result was illustrated over 1200 years ago in the *Book of Creation (Sefer Yetsirah),* written 2nd-8th century in which the act of creation was related to forming possible sequences:

"Two stones [Hebrew letters] build two houses [words], three build six houses, four build twenty-four houses, five build one hundred and twenty houses, six build seven hundred and twenty houses, seven build five thousand and forty houses (4:12)."

Lumpkin (1997) offers a related classroom reproducible activity that extends to the astrology calculations of the 12th-century Spanish scholar Rabbi Abraham ibn Ezra.

Instructors, however, need to help ensure that this does not encourage statements of cultural (or national or religious) superiority. There are sometimes genuine uncertainties about which culture was truly the first to discover something, especially if a more extensive written record survived for some cultures than for others. One should aim to cultivate and model a humble awareness that such claims are always subject to being updated or changed as new scholarship or archaeological findings emerge. And the culture that first explored a mathematical idea may not be the culture that explored it the deepest. In any case, these pitfalls were natural to avoid at this Jewish high school, perhaps because of that school's explicit commitment to pluralism and the presence of non-Jews among the student body, faculty, and staff. Avoiding the pitfalls of insularity (that a more religious day school might be susceptible to), the school supported much intergroup interaction such as an extended visit by the school to a local mosque. Such particular events supported the general goal of ethnomathematics in bridging cultural gaps as well as added rich context to later mathematics classroom explorations (e.g., the Islamic use of tessellations). Kraft (2005) reports the even more inspiring example of Israeli and Palestinian high school students learning and working together at a computer summer camp in the MIT-supported Middle East Education through Technology program (meet.csail.mit.edu).

Cultural Counting

There is cultural significance even in the way a Jewish society marks time. Days of the week are not so much named as numbered (by how they head towards that week's Sabbath day): Sunday is *Yom Rishon* ("first day"), Monday is *Yom Sheni* ("second day"), etc. Like the Muslim lunar calendar, the Jewish calendar has lunar months of 29 or 30 days and the moon is prominent in symbolism and holidays (e.g., *Rosh Chodesh*). Students were further intrigued by how the Jewish calendar is actually luni-solar, having the occasional addition of an extra month (in years 3, 6, 8, 11, 14, 17, and 19 of each 19-year cycle) to retain alignment with the solar year, since a strictly lunar calendar falls 11 days behind a solar calendar each year. This adjustment ensures, for example, that the holiday of Passover always happens during the spring as specified in the Torah (Deuteronomy 16:1).

Also, in Jewish law, an hour is not simply “an hour”, but is $\frac{1}{12}$ the amount of that day’s daylight. Students can use proportional reasoning to verify that at a place and time when the difference between sunrise and sunset times is 13 hours, a Jewish halachic (legal) hour would actually be 65 minutes long. A specific application of this still practiced by many Jews today is determining the proper time or range of permissible times for certain rituals or prayers. For example, the *Shema* prayer affirming God’s oneness must be said by the end of the third halachic hour in the morning. A complete mathematical analysis of the Hebrew calendar assuming no technical knowledge beyond high school algebra is in Gabai (2002), with a more detailed account in Feldman (1978).

It appears that in ancient times, Jews used other numeral systems (such as the Egyptian hieratic and then the Babylonian base 60), before deciding (by the second century) to use the letters of the Hebrew alphabet to represent numerical values assigned to the letters, as the Greeks had already been doing with the Greek alphabet. This numerology is called gematria. (Some examples of Hebrew gematria appeared in the 1998 movie *Pi*, which won the Director’s Award at the Sundance Film Festival. Teachers should be advised, however, that the film is rated R for language and disturbing images.) The 22 letters of the Hebrew alphabet (first letter is *aleph*: א)

א ב ג ד ה ו ז ח ט י כ ל מ נ ס ע פ צ ק ר ש ת

are typically assigned the respective values 1-9, the multiples of 10 from 10-90, and then 100, 200, 300, and 400. An Internet search for "Hebrew numbers" or "gematria" yields more detailed charts, see Appendix. While today's Israeli/Hebrew culture uses the modern Hindu-Arabic base-ten system for most purposes, Hebrew letters are still used for numbers in calendars (days of the month, years within a millennium), and religious books (page numbers; chapters and verses of scripture). Jewish celebrations such as *Lag*(לג) *B'Omer*, *Tu*(טו) *B'Shevat*, and *Tu B'Av* have information about when they occur numerically embedded in their names (like "Cinco de Mayo" or "Fourth of July"). Note that the two letters used in *Tu*(טו) *B'Shevat* correspond to 9+6, instead of the expected 10+5, so as to avoid unnecessarily spelling one of the names of God.

Unlike today's modern Hindu-Arabic decimal system, the ancient Hebrew number system is nonpositional and additive. Therefore, anagrams (words formed by permuting letters) have the same numerical value, thanks to the commutative property of addition! Gabai (2002) gives the example of the Torah's second word, ברא (*Bara*, "He created"), and connects it to one of its (3! = 6) permutations, באר (*Be'er*, "He explained/elucidated"). A gematria-generating mathematical tool that can be used with (or instead of) permutations is partitions: all the ways of writing an integer as a sum of positive integers, not counting the order of addends. For example, there are 5 partitions of 4: 4, 3+1, 2+2, 2+1+1, and 1+1+1+1. Thus, there is a numerical equivalence between the Hebrew words *Gey* (proud; גא;1+3) and *Abbah* (father; אבא ; 1+2+1). An Internet search for "gematria software" turns up several software packages that can find words which are numerically equivalent to each other.

The Hebrew word for life, חי (*chai*), is spelled with the two letters *chet* and *yud*, whose numerical equivalents 8 and 10 sum to 18, which explains the modern Jewish cultural practice of giving (and soliciting) charity in multiples of 18. Speaking of charity, Gabai (2002) notes that the Hebrew word for charity (*tzedakah*; צדקה) is one of the few words whose gematria is equal to its "reverse gematria" (from assigning numbers to the Hebrew alphabet in reverse order), suggesting that this moral value of giving charity "will remain irreversible and unchangeable throughout the person's life" (p. 76). It is interesting to let students deduce the mathematical condition for this – if the n^{th} letter of the Hebrew alphabet is in the word, so must the k^{th} letter, where $k = 22 - n$.

There are variations on how to assign numerical values to the letters, such as using the ordinal numbers 1 – 22 or such as assigning each word the one digit you get when you keep adding up the sum of the digits of its letters' usual numerical values until you have a 1-digit number (e.g., a word with the value 618 would reduce to $6+1+8=15$, which in turn would yield $1+5=6$). In this latter option, students can discover that two words are "linked" if the sums of their letters' numerical equivalents are congruent modulo 9. Such "links" are interpreted in a great variety of ways, ranging from a simple educational mnemonic device (useful for Judaics study) to a mystical, Kabbalistic revelation. Gabai (2002) makes it a point to remind the reader that Jewish mystical and rabbinical writings warn of overuse or misuse of gematria, and it is interesting to encourage students to reflect upon statements such as Gabai (2002):

> From a religious, spiritual, or mystical point of view, we note the repeated appearances of some whole numbers and their multiples, and we wonder about their meaning and significance. From the point of view of logic and mathematics, the repetition of numbers is not unexpected because whole numbers are mentioned so often in the Bible. (pp. 18-19)

The Infinite: How We Count

The Talmudic statement (Mishna Sanhedrin 4:5) that someone who saves a life is viewed "as if he saved an entire world" may be familiar as the tagline of the 1993 Academy Award-winning movie *Schindler's List*. Telushkin (1991) notes that one consequence of this idea of the infinite value of human life is that "saving many lives at the expense of one innocent life is not permitted, since by definition, many ‘infinities’ cannot be worth more than one ‘infinity’” (p. 530). This, of course, is perfectly consistent with arithmetic of transfinite cardinal numbers (e.g., the sum of any countable number of aleph nulls equals aleph null, where aleph null is the cardinality of natural numbers; *aleph*, $\aleph$, is the first letter of the Hebrew alphabet)!

In Jewish tradition, counting people is permitted only in an indirect manner, such as a population census via 1-1 correspondence with the proxy of a coin donation (e.g., Exodus 30:13). A current example of this is a traditional way to see if there is a *minyan* (“prayer quorum”) of at least ten eligible people by reciting a 10-word verse of Torah such as Psalms 28:9, assigning a word to each eligible person present. Interestingly, Zaslavsky (1973) also relates an African taboo on counting people. (As an aside, it is interesting to discuss the extent to which individual dignity is similarly respected by the statistical technique of randomized response (Warner 1965). This survey method asks about personal behaviors or beliefs in a way that gives privacy to potentially embarrassing answers but still allows estimation of the overall answer for the group of people.)

In addition to the idea of infinite value of human life, there may also be a way to apply transfinite arithmetic to *mitzvot* (plural of *mitzvah*). A *mitzvah* is viewed culturally as a “good deed” and religiously as a “commandment”. It is generally accepted that the Torah contains 613 *mitzvot*, including both matters of ritual and matters of ethical behavior. It is tempting to assume that some *mitzvot* are “worth” more than others, and indeed there are seven instances (e.g., *tzedakah*) in which Talmud rabbis refer to a particular mitzvah as equal in value to all the others combined (Donin 1980). While this may seem to be mystifying hyperbole, we can see from the resulting system of mathematical equations that this is perfectly consistent with each *mitzvah* having an infinite value such as $\aleph_0$ (or, technically, a value of 0).

Some Jewish writing (Kaplan 1990; Saks 2002; Saks 1990; Schochet 1979) refers to different levels of infinite spiritual worlds, an idea similar to Georg Cantor's idea that there is an infinite sequence of (mathematical) infinities: $\aleph_0 < \aleph_1 < \aleph_2 < \ldots$. Cantor's demonstration that "the infinite cube contains exactly the same number of points as in the one-inch cube" (Gabai 2002, pp. 41-42) helps us understand "the paradox of God's simultaneous transcendence and immanence" (ibid). Gabai (2002) helps us understand other aspects of the Jewish view of God's presence by referencing Edwin Abbott's classic mathematical novel *Flatland*. One of God's many names in Judaism is *Ein Sof*, which was introduced in the 13th century and has been translated as "without end" or "infinity".

Connections to Content: Exponential and Logarithmic Functions

While I was teaching at the high school, an article from the local paper (Copans 2003) gave my precalculus students a real-life vehicle to discuss and apply the carbon dating model from their book (Larson, Hostetler, Edwards 2001): the ratio of C_{14} to C_{12} in dead organic material equals $10^{-12}e^{-t/8223}$. The newspaper story discussed how carbon dating confirmed that a stone tablet detailing repair plans for Solomon's temple contained writing from the 9th century BCE. The class verified that this would be consistent with a measurement of about 70.5% of the radioactive carbon 14 (whose half-life is 5700 years) remaining. This Biblical archaeology context appeared to engage the class far more than did a textbook carbon dating problem about a generic fossil, because of the Judaics connection, not out of any religion versus science tension. (Indeed, evolution and big bang theories were taught at this school and can be viewed in a way that is consistent with Jewish religious teachings.)

Another day, after the precalculus students had returned from a school field trip to see a *mikvah* (a ritual immersion pool), I had them explore an excerpt from Talmud (Mikva'os 7:2) that required not only applying logarithms, but also examining modeling assumptions: “If there were exactly forty *se’ahs* [of suitable water] in a *mikvah*, and a *se’ah* [of unsuitable liquid] was put in, and then a *se’ah* [of the resulting mixture] was removed, the *mikvah* is still valid.” A *se’ah* is about 8.25 liters. This excerpt from Littman (1989) provided a vehicle to explore how many times this process can be repeated until the majority of liquid is unsuitable, and to discuss what modeling assumptions might be made about how well the liquids mix. In particular, what is the answer if a *se’ah* of uniform mixture is removed each time and what is the answer under the “worst-case scenario” that a *se’ah* of suitable water is removed each time? Littman (1989) contains a detailed analysis of this and other Talmudic examples, including a maximization problem in geometry and how the Euclidean algorithm applies to calculating the occurrences of the new moon.

Value of Pi

A value of π can be obtained from I Kings 7:23:

“He made the ‘sea’ of cast [metal] ten cubits from its one lip to its [other] lip, circular all around, five cubits its height; a thirty-cubit line could encircle it all around.”

It appears the value of π implied here is simply $\frac{30}{10}$ (an error of 4.5%) until a student asks if we need to consider the tank’s thickness -- given three verses later as one-handbreadth, so the inner diameter is 10 cubits minus 2 handbreadths. (Of course, this is also a chance to discuss issues of measurement!) Using the Talmudic value of $\frac{1}{6}$ cubit for one handbreadth, the inner diameter becomes $9\frac{2}{3}$ cubits and dividing 30 by $9\frac{2}{3}$ yields more accuracy (error: 1.2%). Applying a more subtle and technical approach to I Kings 7:23 (see Posamentier & Lehmann 2004 or Tsaban & Garber 1998), the ratio of gematrias for the written and spoken forms of a key Hebrew word (for “line”) in that verse is 111/106, which when multiplied by 3 yields a very refined approximation for π: $\frac{333}{106}$ (error: 0.0026%). Very few words in the Torah have different oral and written forms.

Geometry Connections

Like many cultures, Jewish culture has a very prominent role for food. Bodner (2001) offers a rich variety of geometry formulas applied to the matter of assessing whether various shapes (e.g., rectangular prism, sphere, cylinder, torus) of food are of sufficiently large volume to allow and require the saying of a particular post-meal blessing. This threshold unit of volume is one *k'zayis*, which is about the size of an olive or half an egg.

A more mainstream example is exploring the geometry of the Star of David, which has become a universally recognized symbol of Jewry and appears on the Israeli flag. My geometry students enjoyed how the Star of David appears in the geometry construction activity of Lide (2004) and how the sum of its 24 interior angles turns out to be a very nice multiple (100) of *chai*. When we discussed fractals, students were interested that "stage 2" of the particular fractal known as the Koch snowflake (the interested reader is referred to www.shodor.org/interactivate/activities/koch/) is a Star of David. And more profoundly, fractal self-similarity has been used as a metaphor for thematic microcosms in Jewish text. For example, Ellis (1997) states:

> The focus in *parashah* Be-har [Leviticus 25:1-26:2] on Shabbat, on the giving of the land, on ethical behavior, on living in peace and love with our sisters and our brothers, on living in active support of our sisters and our brothers: this is the focus of the entire Torah….Line by line, breath by breath, stem by stem, blossom by blossom, each verse of our *parashah*, like each verse of Leviticus, mirrors fractal-like the glorious and transcendent vision of the whole. (p. 34)

When the *sukkah* (temporary hut-like dwelling from the Biblical holiday of Sukkot that non-Jews may know by the name Feast of Tabernacles) was built on the school campus grounds, we had a “geometry treasure hunt” to find various types of angles and polygons amidst the beams and poles.

I had students explore various geometric shapes and concepts through origami constructions of various Jewish cultural symbols and holiday objects (Temko 1991, 1992, 1994), such as a ram’s horn (for the Jewish New Year) or a Star of David. One test problem referenced shapes and lengths from diagrams of the ancient Temple (Wigoder 1974).

Also, examples of geometry in the Talmud include: using the circumference to calculate a circle’s diameter (Eruvin 13b, 14a, 76a-b; Succah 7b), calculating the area of a square’s circumscribed or inscribed circle (Eruvin 76a-b; Succah 8a-b), using a side to find a square’s diagonal (Eruvin 57a, 76b; Succah 8a), and using ratios to find faraway objects’ heights (Eruvin 43b).

Structural Parallels between Judaism and Mathematics/Logic

With my geometry class, I discussed the observation by Gabai (2002) that Judaism can be viewed as a deductive system with the fundamental undefined term of God, the Torah contents as axioms, and the Talmud as theorems derived from axioms. Davis & Hersh (1981) make an interesting juxtaposition with the axiom that an infinite set exists and the “axiom of God as presented by Maimonides (Mishneh Torah, Book 1, Chapter 1): ‘The basic principle of all basic principles and the pillar of all the sciences is to realize that there is a First Being who brought every existing thing into being.’” Davis and Hersh (1981) also quote Maimonides as giving a “proof by contradiction (p. 118)” of that assertion of God’s existence (“for if it be supposed that he did not exist, then nothing else could possibly exist”) and quote Saddia Gaon (10th century theologian and leader of Babylonian Jewry) as supplying an indirect proof of God’s uniqueness:

“For if He were more than one, there would apply to him the category of number, and he would fall under the laws governing bodies.”

During our chapter on deductive reasoning and logic, I supplemented problems from the book by having them critique reasoning using Jewish content such as: “If someone is a Jew, then she believes in the Torah. If someone believes in the Torah, then she observes the Sabbath. Therefore, if someone is a Jew, then she observes the Sabbath.” On another assignment, students were asked to construct a logical argument to respond to a not-uncommon proselytization attempt (by a “Jew for Jesus” named Mitch), which had been the focus of a recent school assembly. One 9th-grade student wrote:

“If Jesus is not the Messiah, then Mitch is wrong. If Jesus is the Messiah, then he would have built the 3rd Temple in Jerusalem, bring all the Jews to Israel, bring world peace and worldwide recognition of God. There is no 3rd Temple or world peace. There are atheists and Jews living outside Israel. Therefore, Jesus is not the Messiah (Modus Tollens). Therefore, Mitch is wrong (Modus Ponens).”

Another example was to decide if the statement “If an animal chews its cud, then it is kosher” is true – and also assess the validity of its converse. (Note: a kosher animal chews its cud *and* has split hooves.) While formal data was not collected, there was anecdotal evidence that students found these examples among the most memorable for the course.

Resources such as Sion (1997) make explicit connections between logical argument structures in Jewish text (e.g., “*qal vachomer*”, meaning ‘lenient and stringent’ and often referred to as ‘all the more’) and their secular Aristotelian counterparts (*a-fortiori ratione*, Latin; meaning ‘with stronger reason’). Sion (1997) gives the example of the first occurrence of *qal vachomer* in the Torah (“and perhaps historically, in any extant written document”) in Genesis 44:8, paraphrasing the argument as:

You will agree to the general principle that more honesty is required to return
found money than to refrain from stealing a silver goblet.
We were honest enough to return found money.
Therefore, you can trust we were honest enough to not steal the silver goblet.

Reflections

While I was teaching at the Jewish high school, I had the opportunity to reflect not only on content, but also on the process of teaching. I was struck by parallels between Jewish views of creation and the active, student-centered pedagogy supported by NCTM (2000). As Kanter (1997) explains:

Through an act of *tzimtzum* [contraction], the teacher allows the child to be actively engaged and to learn by doing (*na'aseh v'nishma*). Through that same act, the teacher acknowledges that children are now and will more fully become images of G-d (*tzelem elohim*), each in his/her own unique way (*ein domeh l'haverio*)" (p. 45).

And I also gained more appreciation for Rabbi Hanina's statement in the Talmud (Taanit, 7a): "Much have I learned from my teachers, more from my colleagues, but most from my students."

Even though there are few Jews among the inservice and preservice teachers I now work with in my university position, I find many aspects that transfer or transcend. First, by sharing some of my experiences with integrating Jewish mathematics, I can serve as a model to give teachers a tangible sense for the process and power of making similar connections between mathematics and their own cultures (and the cultures of the students they will teach). Second, every culture has distinctive mathematical contributions to discuss and interesting quotations from respected figures that can be posted, etc. (For example, in my recent work with a predominantly Hispanic middle school, I facilitated the posting of Latino/a biographies from the SACNAS Biography Project, http://64.171.10.183/biography/listsscientist.asp.) Also, techniques and issues of topics such as numerology can be applied to and critically discussed with respect to any culture's writings. It is not hard to imagine that making such connections could help one teach from a more integrated self and also do a better job of teaching “the whole child”. Cultural mathematics is a powerful way to build community on many levels, which can spill over to a more receptive and supportive classroom culture even when talking about mathematics that may not (yet) be connected to culture.

Students (and most faculty, for that matter) at the Jewish high school had typically viewed Judaics and secular subjects as compartmentalized experiences. While the school’s literature claims that the school’s Judaic studies are “integrated into the general studies curriculum”, this had occurred only in isolated instances and only in humanities classes such as history and literature. Sion (1997) asserts that integration across disciplines is

…a fundamental need of the human mind and culture; the alternative of compartmentalization involves a sort or mental or cultural forcing, and can only be a temporary measure. Conflicts between experience and reason, on the one hand, and faith, on the other, can best be diffused by a dispassionate consideration of the underlying logical issues. Many disagreements can be harmonized, by showing at the least that the sides are simply alternatives in a disjunctive proposition, which though they may inductively be of varying probability are deductively on equal footing, for all practical purposes a matter of personal choice. (p. 216)

By bridging the Jewish and secular worlds in the mathematics classroom, I noticed a broadening of their view of and their appreciation of mathematics. This felt important, considering that many students (and faculty) at the school had not had a favorable or robust view of mathematics beyond skills that had to be learned to succeed on standardized tests required for college admission. A possible consequence of giving students the experience of mathematics as culture is to allow them to see that mathematics, like culture, can be fallible. Sion (1997) and Littman (1989) both discuss some apparent imperfections of logic or computation in the Talmud. This supports more of a progressive educator (or even public educator) ideology, as described by Ernest (1991), and arguably encourages students to become more active and inquisitive in their engagement with mathematics.

As a secondary byproduct, it was interesting that there may have also been a broadening and appreciation of their Judaism as well. This may be enthusiastically welcomed in light of the recent study commissioned by the pluralistic Jewish youth group B'nai Brith Youth Organization in which Teenage Research Unlimited surveyed 1,153 respondents aged 10-18 (of all religions) about their attitudes towards religion (Weiss 2006). It was found that "92% of the teenagers who view religion and faith as an important part of their lives say they want a better connection and 52% of these teens are looking for new and unconventional ways to connect to their religion." (BBYO 2006). Ethnomathematics has appeared to be useful in some cultures as a component of cultural revitalization. The Jewish outreach organization Aish HaTorah (http://www.aish.com/) has been known to include in its "Discovery Seminar" presentations the equidistant letter sequence "Bible codes" à la the best-selling Drosnin (1997). It should be noted, however, that there have been Jewish and scientific critiques of these codes (Simon 1998, McKay, Bar-Natan, Bar-Hillel, et al. 1999), and these can be discussed in more advanced classes.

Some Pedagogical Considerations

As with any area of application or integration, teachers who include culturally relevant mathematics in their assessments should ensure that the presentation is sufficiently self-contained so that students with more culturally-specific knowledge do not have an undue advantage. This was usually done by providing the necessary context or by having the context closely parallel standard textbook scenarios. For example, one of my geometry assessments of this type closely followed their textbook (Schultz, Hollowell, Ellis, et al., 2001) "archaeologist" scenario of estimating the size of an ancient plate by finding the circumcenter from a surviving outer fragment of the plate. The context for my version was the Jewish engagement ritual (*tana'im*) of the couple's mothers' breaking of a plate (the fragments of which are sometimes offered by the bride to unmarried relatives for good luck). As Kaplan (1983) explains:

> The reason for breaking the dish is to show that we mourn for Jerusalem and other martyred Jews even at the height of our joy. A china dish is broken to show that, just as a china plate can never be fully repaired, a broken engagement is an irreparable breach. Even if the bride and groom are later reconciled, the breaking of the formal Tanaim contract is considered very reprehensible. (p. 28)

Many topics in Jewish culturally relevant mathematics are rich enough to explore at multiple points on the curriculum ladder. For example, the children's Hanukkah game of spinning a *dreidl* (4-sided top) yields an equiprobable experiment for Grades 1-6 (Krause 1983), but can also yield more sophisticated probability analyses all the way to the derivation (suitable for a first university course in probability) of the expected payoff on the n^{th} spin with N players (that shows the usual game is biased in favor of the first player) and what rule changes would make the game "fair" to all players (Feinerman 1976; Trachtenberg 1996). Also, there are many levels to use concepts of measurement (whose importance is reflected in Leviticus 19:35-36), such as how one might estimate the volume of Noah's Ark given statements such as Genesis 6:15.

Jewish culture has been greatly influenced not only by Biblical times, but also modern world history. Wainer (1994, 2004) shows graphs that could supplement a (Judaics or history) unit on the Holocaust, giving striking context to the three square feet of space each prisoner of Belsen Concentration Camp was allotted, for example. For that matter, students may be moved to learn how the Nazi WWII experiments on prisoners led to the creation of research ethics codes that now protect all of us (Lesser and Nordenhaug 2004). And Shulman (2002) cites two further discussion-provoking examples: how Nazi ideology infiltrated even German mathematics textbook problems at the time and subsequent controversy about whether already-collected Nazi "research data" should be used if it might save lives. Yet another kind of ethnomathematics on this topic relates to the 2004 documentary movie *Paper Clips*, in which middle school students in rural Tennessee decide to collect six million paper clips in an attempt to grasp the concept of six million Jewish Holocaust victims. The students and their community are transformed by the process, during which they meet Holocaust survivors from around the world (Schroeder & Schroeder-Hildebrand, 2005).

It is hoped that this discussion of a rich variety of examples and issues associated with Jewish culturally relevant mathematics may inform and inspire teachers and curriculum writers of all backgrounds.

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APPENDIX: Hebrew Alphabet and Numerical Equivalents

Source: www.omniglot.com/writing/hebrew.htm

ט	ח	ז	ו	ה	ד	ג	בּ/ב	א
טֵית	חֵית	זַיִן	וָו	הֵא (הֵי)	דָּלֶת (דָּלֶד)	גִימֶל	בֵּית/בֵית	אָלֶף
tet	chet	zayin	vav	he(y)	dalet(d)	gimel	bet/vet	alef
t	h/ch	z	w	h	d	g	b/v	'
[t]	[x]	[z]	[v]	[h]	[d]	[g]	[b, v]	[ʔ, ∅]
9	8	7	6	5	4	3	2	1
ס	ן	נ	ם	מ	ל	ך	כּ/כ	י
סָמֶךְ	נוּן סוֹפִית	נוּן	מֵם סוֹפִית	מֵם	לָמֶד	כָף סוֹפִית	כָּף/כָף	יוֹד (יוּד)
samech	nun	nun	mem	mem	lamed	kaf	kaf/khaf	yod (yud)
s	sofit	n	sofit	m	l	sofit	k/kh	y/i
[s]	final nun	[n]	final mem	[m]	[l]	final kaf	[k, x]	[j]
60		50		40	30		20	10
ת	שׁ/שׂ	ר	ק	ץ	צ	ף	פּ/פ	ע
תָּו/תָו	שִׁין/שִׂין	רֵישׁ (רֵיִשׁ)	קוֹף (קוּף)	צָדִי סוֹפִית	צָדִי (צָדִיק)	פֵא סוֹפִית	פֵּא	עַיִן
tav(f)	sin/shin	resh (reyish)	kof (kuf)	tzadi	tzadi(k)	pe	pe(y)/fe	ayin
t	sh/s	r	k	sofit	tz/ts/z	sofit	p/f	'
[t]	[ʃ, s]	[ʁ]	[k]	final tzadi	[t͡s]	final pe	[p, f]	[ʔ, ∅]
400	300	200	100		90		80	70