HISTORIA MATHEMATICA **25** (1998), 75–84
ARTICLE NO. HM972185

On the Rabbinical Approximation of π

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We discuss the Rabbinical tradition of geometry concerning circular shapes, as it appears in the Babylonian Talmud and in later commentaries. Three explanations of the difference between π and the Rabbinical value for it, so far not widely known among the scientific community, are given. © 1998 Academic Press

Nous discutons ici la tradition rabbinique en géométrie à propos des figures circulaires, telle qu'elle apparaît dans le Talmud babylonien et dans des commentaires ulterieurs. Trois explications sont proposées pour la différence entre π et la valeur donnée dans la littérature rabbinique. Ces explications semblent peu connues dans la communauté scientifique. © 1998 Academic Press

Wir diskutieren die rabbinische Tradition der Geometrie der kreisförmigen Formen, wie sie im babylonischen Talmud und in späteren Kommentaren erscheint. Wir geben drei Erklärungen für den Unterschied zwischen π und seinem rabbinischen Wert, die unter Wissenschaftlern noch nicht weit bekannt sind. © 1998 Academic Press

MSC 1991 subject classifications: 01A35, 01A17.
Key Words: nonstandard geometry; history of pi; Jewish mathematics.

1. INTRODUCTION

The *Talmud*, which literally means study, is a monumental Hebrew work consisting of knowledge accumulated over thousands of years through extensive study by Jewish scholars. The Talmud consists of two portions: the *Mishna* and the *Gemara*. The teaching contained in the former was transmitted from generation to generation by word of mouth, and finally compiled and edited by Rabbi Yehuda Hanasi (=the President) at the end of the second century C.E. It is divided into six sections, each divided into tractates (or treatises) which are subdivided into chapters. Each chapter is divided into paragraphs.[1] The Gemara consists of discussions and disputations on the Mishna. This induces a division of the Talmud, according to the tractates of the Mishna. Those taking part in the discussions are called *Amoraim* (singular: *Amora*), meaning tellers or interpreters. It is common to say 'the Gemara' (says, asks, etc.) when referring to an anonymous Amora who is quoted in the Gemara. There are two schools of Amoraim: the Babylonian and the Palestinian. Each school compiled its own Talmud: the *Babylonian Talmud* and the *Palestinian* (or *Jerusalem*) *Talmud*, respectively.

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[1] Thus, e.g., Mishna Ohalot XII 6 means Mishna, Tractate Ohalot, Chapter XII, sixth paragraph (Tractate Ohalot is in Section Teharot, but the section is usually omitted).

0315-0860/98 $25.00

Small portions of the Babylonian Talmud began to be published soon after the introduction of printing. The first complete Talmud[2] was printed by Daniel Bomberg in Venice between 1520 and 1523 C.E. This *editio princeps* determined the external form of the Talmud for all time, including the pagination and the running commentaries of Rashi[3] and the *Tosafot*.[4] In this printing, the Talmud is divided into folios, each of which consists of two pages.[5] The best-known among more modern editions of the Talmud is the one printed in Vilna by the widow and the brothers of the printer Romm in 1880 C.E. This edition is still the most popular edition among Jewish Talmud scholars.[6]

Rabbi Yohanan Ben Nappaha (="son of the blacksmith") (*ca*. 180–*ca*. 279 C.E.) was one of the greatest Amoraim. Rabbi Yohanan lived in Israel, and his teachings comprise a major portion of the Palestinian Talmud. He is also quoted more than 4,000 times in the Babylonian Talmud. In addition to his knowledge of religious law (*Halacha*), he mastered mysticism (Talmud Hagiga 13a), the science of intercalating months (Palestinian Talmud, Rosh-Hashana 2:6), medicine (Talmud Shabbat 109b; 110b), mathematics,[7] and other sciences [12, 10: 144–147].

2. THE BIBLICAL AND TALMUDIC APPROXIMATION OF π

The Rabbinical approximation of π is discussed[8] in the Babylonian Talmud, Eruvin 14a. The Mishna there states the rule "Every [circle] whose circumference is three handbreadths, is one handbreadth wide"[9] (hence the ratio of the circumference of a circle to its diameter is taken to be $\pi_0 = 3$). The Gemara asks "Where is this learned from?" Rabbi Yohanan gives the Biblical authority—the verse 1 Kings 7:23—"And he made a molten sea [tank], ten cubits from one brim to the other. It was round all about, and its height was five cubits. And a line of thirty cubits did circle it round about." The Gemara argues "But it had a brim," that is, the diameter perhaps was measured from outside, while the circumference was measured from inside, and therefore the given value does not represent π. Rabbi

[2] Unless otherwise indicated, "Talmud" always refers to the Babylonian Talmud.

[3] "Rashi" is the acronym of Rabbi Shlomo Itshaqi of Troyes (1040–1105 C.E.).

[4] The word "Tosafot" means "addenda." This commentary was written mainly by Rashi's sons-in-law and grandsons during the 12th and 13th centuries C.E.

[5] Thus, e.g., Talmud Suca 8a means (Babylonian) Talmud, Tractate Suca, Folio 8, first page. The Palestinian Talmud, on the other hand, is referred to by the tractate, the chapter, and the paragraph number. Palestinian Talmud, Rosh-Hashana 2:6 thus refers to the sixth paragraph of the second chapter in tractate Rosh-Hashana of the Palestinian Talmud.

[6] A good reference on the Talmud is [12, 15: 750–779].

[7] The pertinent references are: Talmud Eruvin 14a and 76a; Suca 7b; Menahot 97b–98a; Palestinian Talmud Kilaim 2:8 and 5:3; Midrash Kohelet Raba Chapter 12, first section; etc.

[8] Of course, the symbol π was not used in the early Rabbinical literature. The number π was usually referred to indirectly, or by "the ratio of the circumference of a circle to its diameter," "the number which when multiplied by the diameter produces the circumference," etc.

[9] This rule also appears in the Talmud, Eruvin 76a and Suca 7b, etc. A variation of this rule appears in Mishna Ohalot XII 6.

Papa suggests that the brim was very thin, therefore negligible. Again, the Gemara objects: "But there is still a slight [thickness]," so the value 3 given above would not describe the ratio of the circumference to the (whole) diameter. Therefore, the Gemara concludes, both the circumference and the diameter given in the verse refer to the inner side of the tank, as otherwise the Mishna would not have stated the rule as is.

This might seem very surprising [1], knowing that the ancient Babylonians and Egyptians used better approximations long before the verse 1 Kings 7:23 was written.[10] We see that the Gemara insists on learning the ratio $\pi_0 = 3$ from the Bible, as an *exact* parameter in calculations for religious purposes. Moreover, Rabbi Yohanan does not answer, "This is a mathematical fact," nor does he say "One can check this via measuring" because it is known that the value is not mathematically correct.[11] Hence, he answers that it is written in the Bible, telling us that we should use it for religious purposes, regardless of its being mathematically correct or not.

Another geometric rule given in the Talmud (Eruvin 56b, 76a; Suca 7b) is "How much is the square greater than the [inscribed] circle? A quarter," that is, the circumference of a circle inscribed in a square is a quarter less than (or 3/4 of) the perimeter of the square. There is a corresponding areal rule (Eruvin 76b, Suca 8a), saying that this is the case with the ratio of the areas as well: "A circle in a square—a quarter." These rules are immediate consequences of the usual geometrical rules and the approximation $\pi_0 = 3$.[12] In [29; 30], we discuss a proof which derives the areal rule from the rule for the circumference using infinitesimals.

It is interesting to check whether more precise values were known to the ancient Hebrews. The answer to this may be found in the Hebrew Bible [21; 22]. There is a Rabbinical tradition on the reading-versus-writing disparity in 1 Kings 7:23. According to Hebrew scriptural tradition, the word meaning "line" is written as קוה, but read as קו. This is exactly the case with the values for π. Even though we see (via measuring or mathematical proof) a more precise value for π, call it π_H, the Hebrew tradition tells us to use the value 3 (for religious purposes). In gematria,[13]

[10] The dating of this verse is ambiguous. It was written after *ca.* 965 B.C.E. (when Solomon became a king), but not much later than 561 B.C.E. [12, 8: 766–777; 12, 10: 1030].

[11] In *Mishnat Ha-Midot*, the value $3\frac{1}{7}$ is used for π. According to [6, 12; 12, 11: 1121–1124; 26, 208–209], this text dates to the second century C.E., and shows that the value $3\frac{1}{7}$ was known to the Jewish sages at that time. Gad B. Sarfatti in [24; 25] argues for a redating of *Mishnat Ha-Midot* to between 850 and 1200 C.E., but see [14, 156; 23] for certain doubts concerning this redating.

[12] Indeed, these rules are equivalent to the rules $A = \pi r^2 = \frac{\pi}{4} \times d^2$, $P = \pi d = \frac{\pi}{4} \times 4d$, where d^2 is the area of the square, $4d$ is its perimeter, and π is taken to be 3.

[13] *Gematria* (from the Greek *γεωμετρία*) is a mathematical method which uses letters to signify numbers (for example in Hebrew, א = 1, ב = 2, etc.). Words have the numerical value which is the sum of the numerical values of their letters [1; 12, 7: 369].

this is expressed in the equation of the ratios $\frac{\pi_H}{3} = \frac{\text{קוה}}{\text{קו}} = \frac{111}{106}$, whence $\pi_H = 3 \cdot \frac{111}{106} = 3 + \frac{15}{106} = \mathbf{3.1415}094\ldots$, while $\pi = \mathbf{3.1415}926\ldots$.[14]

Why, then, not use a more precise value? Maimonides,[15] in *Perush Ha-Mishna* (his commentary to the Mishna), Mishna Eruvin I 5, states the irrationality of π:

> You need to know that the ratio of the circle's diameter to its circumference is not known and it is never possible to express it precisely. This is not due to a lack in our knowledge, as the sect called Gahaliya [the ignorants] thinks; but it is in its nature that it is unknown, and there is no way [to know it], but it is known approximately. The geometers have already written essays about this, that is, to know the ratio of the diameter to the circumference approximately, and the proofs for this. This approximation which is accepted by the educated people is the ratio of one to three and one seventh. Every circle whose diameter is one handbreadth, has in its circumference three and one seventh handbreadths approximately. As it will never be perceived but approximately, they [the Hebrew sages] took the nearest integer and said that every circle whose circumference is three fists is one fist wide, and they contented themselves with this for their needs in the religious law [13; 20].

Maimonides' statement is one of the earliest extant ones making that claim.[16]

Matityahu Hacohen Munk [21; 22] suggests a mystical explanation: some of the geometrical rules did not hold in King Solomon's temple, according to Hebrew ancient traditions (see, for instance, Talmud Megilla 10b; Yoma 21a; Baba Batra 99a [7; 8; 28]). In the temple, the ratio of the circumference of a circle to its diameter was exactly π_0.[17] In our reality this fails, but in order to join our reality with the

[14] Unfortunately, there are no known references to this exegesis in literature earlier than [21]. (Medieval commentators used the values $3\frac{1}{7}$ or "a little less than $3\frac{1}{7}$" for π.) It is possible that Matityahu Hacohen Munk [21] was the first to note this fact. It is impossible to answer the question whether the above exegesis is the reason for the disparity or not (see also [1]), though there is some traditional evidence in favor of Munk's explanation; see [8]. Here, we shall give an analysis suggesting that this value was indeed known to Rabbi Yohanan. It is interesting to mention, in this context, another ancient gematria: an inscription of Sargon II (727–707 B.C.E.) states that the king built the wall of Khorsabad 16,283 cubits long to correspond with the numerical value of his name [12, 7: 369].

[15] This is Rabbi Moshe (=Moses) Ben Maimon, acronym Rambam (1135–1204 C.E.), whose Arabic name was Ibn al-Maimūn. He was said to be "The greatest Moses after the first Moses."

[16] Various ancient Greek writers, including Hero, Eutocius, and Simplicius, understand the difficulty of finding an exact value for the ratio, and seem to realize the possibility of its being irrational [17], yet it appears that none of their extant statements are as strong as Maimonides' [15]. See also [16, 363–364]. As for medieval mathematicians preceding Maimonides, we have the following: Yūsuf al-Mu'taman (11th century C.E.), in the *Istikmāl* (which was revised and taught by Maimonides), cites π in the chapter dealing with irrationality [9, 247]. However, he does not explicitly assert such suspicions. The only explicit statement concerning the irrationality of π in the earlier extant literature is to be found in al-Bīrūnī's Masudic Canon (*ca.* 1030 C.E.) (*Qānūn al-Mas'ūdī*, Book III, Chap. 5): "and the number of the circumference has also a ratio to the number of the diameter, although this (ratio) is irrational" [4, 217; 10]. It is not known whether Maimonides knew the Masudic Canon [11]. Anyway, the irrationality of π was proved (by Lambert) only in the 18th century. It is therefore still a mystery what made Maimonides so sure about the irrationality of π. Victor J. Katz [15] has noted that this is similar to Ptolemy's claim (in *Almagest* I, 10) that one cannot trisect an angle using a straightedge and a compass: "The chord corresponding to an arc which is one-third of the previous one cannot be found by geometrical methods" [27, 54].

[17] For geometric and physical models where circles may have this property, see [28].

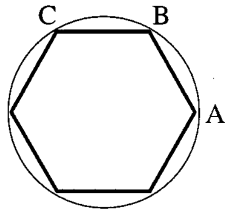

FIGURE 1

"world of truth,"[18] the temple's values should be used in calculations for religious purposes.

Of course, applying the halachic π_0 naively to our reality would yield circles which do not satisfy the halachic requirements. For example, in order for a circle (in our reality) to circumscribe a certain square, its circumference must be π times the diagonal of the square. Using π_0 yields a circle too small.

Nevertheless, even in our reality, it is possible to "experience" π_0 in a manner of speaking. This is accomplished if one computes the circumference not of the circle but rather of the regular hexagon inscribed in it. Then, the circle circumscribing this hexagon will satisfy the halachic requirements; it will circumscribe the square in the above example.

Rabbi Haim David Z. Margaliot [19] noted this possibility more than two decades before Munk.[19] He suggests that the reason for this was that the circumference of the circle was measured from inside using a stick[20] of length equal to the radius of the needed circle. In his interpretation, one edge of the stick was placed at an arbitrary point A on the circle, and the other edge was used to find the point B on the circle. Then the edge was put on B in order to find C, etc. (see Fig. 1). If, after six iterations, the stick's edge returned to the point A, then the "circumference" of the circle was six times the length of the stick or π_0 times the diameter.[21]

Similarly, when the halachic requirement is on the area of the circle, the calculation involving π_0 is applied to the inscribed regular dodecagon [19; 21; 22].[22]

Rabbi Shimon Ben Tsemah (1361–1444) suggests another explanation in *The Tashbets* (Part I, Responsa 165): in fact, more precise values for π were known to the Talmudic Rabbis, but in order for their students to understand, they used the less

[18] For Isaac Newton, "The temple of Solomon was the most important embodiment of a future extramundane reality, a blueprint of heaven; to ascertain every last fact about it was one of the highest forms of knowledge, for here was the ultimate truth of God's kingdom expressed in physical terms" [18, 162] (quoted in [1; 3]).

[19] He did not, however, suggest the idea of alternative geometry in the Temple.

[20] It would have been difficult to measure from inside using a rope.

[21] If there was an overlap, then the circle was considered to have a smaller circumference, and in the remaining case, it had a larger circumference.

[22] We have found no reasonable justification for this claim (apart from the fact that the dodecagon satisfies the halachic formula for the area).

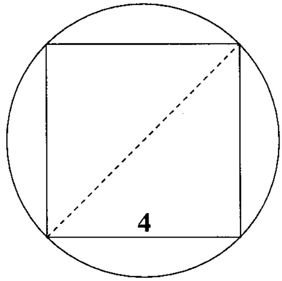

FIGURE 2

precise value—“One should always teach his student in the easiest way” (Talmud Pesahim 3b; 63b). However, *de facto* they used more precise values. In order to understand this, we have to introduce the relevant parts from the discussion held in Talmud Suca 7b–8b.[23]

Relying on Rav’s[24] rule that “A [square] booth (or tabernacle) less than 4 by 4 cubits is unfit,” Rabbi Yohanan said: “A booth built in the form of a kiln (that is, circular) whose circumference is long enough to seat 24 persons is fit for use; if not, it is unfit.”[25] Knowing that one person occupies one cubit by one cubit, the Gemara finds the minimal circumference of a circular booth sufficiently large to contain a square of side 4 cubits.

The diameter of the booth is the diagonal of the square, which is—according to the rule “Each handbreadth in a square is $1\frac{2}{5}$ handbreadths in its diagonal”[26]—$4 \cdot 1\frac{2}{5} = 5\frac{3}{5}$ (see Fig. 2). Hence, the circumference is $5\frac{3}{5}\pi_0 = 16\frac{4}{5}$. Rabbi Assi provides an explanation of Rabbi Yohanan’s statement: the 24 persons should sit *outside* the booth (see note 25), as shown in Fig. 3, where each section corresponds to the space occupied by one person. The circumference of the circle circumscribing the persons is, according to Rabbi Yohanan’s statement, 24 cubits; therefore its diameter is $24/\pi_0 = 8$ cubits. As the diameter of the booth is 2 cubits (one from each side) less than the diameter of the outer circle, we conclude that the diameter of the booth is 6 cubits.[27] Rabbi Yohanan thus gives us an ingeniously practical method, understandable even to the mathematically illiterate person, to check that the booth

[23] For a comprehensive discussion, see [7; 8]. Here, we follow the presentation of [5].

[24] Rav (third century C.E.) was a leading Babylonian *Amora* and founder of the academy at Sura. His name was Abba Ben Aivu. He is generally known as Rav since he was the “teacher (Rabbi) of the entire diaspora” (Talmud Betsa 9a, and Rashi thereto) [12, 13: 1576].

[25] Note that the booth is not intended for the use of the 24 mentioned persons. The statement only gives a way to estimate the circumference of the booth.

[26] Hence $\sqrt{2}$ is taken to be $1\frac{2}{5}$. This value is presented in Talmud Eruvin 76a; Suca 8a, etc.

[27] Here we must use the fact that the space occupied by a person is flexible and may be less than one square cubit. This may be the reason why Rabbi Yohanan uses the term “persons” instead of “cubits.”

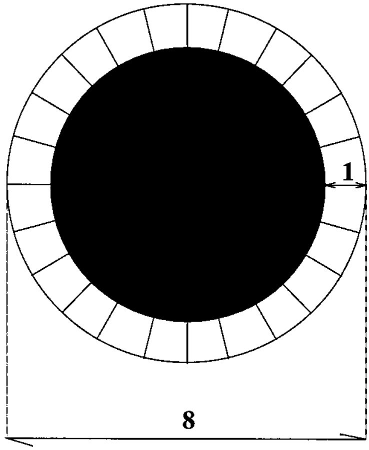

FIGURE 3

has a circumference of 18 cubits.[28] As 18 cubits is more than the minimum ($16\frac{4}{5}$ cubits) required, it seems that Rabbi Yohanan did not mind being somewhat inexact.

However, the following problem now arises:[29] Rabbi Yohanan's words "if not, it is unfit" suggest that he was very precise in his statement. Moreover, Rabbi Yohanan said (Talmud Shabbat 145b)[30] "If it is as clear as day, say it; if not, do not say it." If indeed Rabbi Yohanan used the inexact values, he could have said that 23 persons suffice. This would give $(23/\pi_0 - 2)\pi_0 = 17$ cubits for the circumference of the booth, which is much closer to $16\frac{4}{5}$ and yet more than the minimum requirement.

The solution to this problem is to be found in Rabbi Shimon Ben Tsemah's explanation, which is as follows. Rabbi Yohanan's statement is quite precise, if we assume that he used more precise values for π and $\sqrt{2}$.[31] For this, he takes $3\frac{1}{7}$ for π and d "slightly greater than $1\frac{2}{5}$" for $\sqrt{2}$. The minimum circumference is (see Fig. 2) $4 \cdot d \cdot 3\frac{1}{7}$ which is a little more than $17\frac{3}{5}$. The circumference of the booth is (see Fig. 3) $(24/3\frac{1}{7} - 2)\ 3\frac{1}{7} = 17\frac{5}{7}$, which is more than the minimum $17\frac{3}{5}$ and the difference is not more than $\frac{4}{35}$ cubits.

[28] We shall soon see that the situation is much more complicated, and Rabbi Yohanan's method elegantly bypasses these complications.

[29] Other problems, which are beyond the scope of this paper, also arise (see [2]). However, the solution we present here is just as good for the problems which are not discussed here.

[30] This concerns the verse (Proverbs 7:4) "Tell the wisdom: 'Thou art my sister'."

[31] The inaccuracy of the value $1\frac{2}{5}$ is proved in the Tosafot commentary (see note 4) on Talmud Suca 8a. The proof is as follows: take a square of side 10 cubits, and join the central points of its sides to form a square of area half of the original's or 50 squared cubits. According to the above rule, the side of the new square is $5 \cdot 1\frac{2}{5} = 7$ cubits, hence its area is 49 squared cubits, a contradiction. This proves that $5\sqrt{2} > 7$ or $\sqrt{2} > \frac{7}{5} = 1\frac{2}{5}$.

Of course, we do not intend to claim that Rabbi Yohanan knew the exact numerical values for π and $\sqrt{2}$. Yet we suggest that Rabbi Yohanan may have known the value π_H given in the above exegesis.[32] We begin by reversing the computation of the circumference circumscribing the square.

Suppose $\sqrt{2}_R$ is an approximation of $\sqrt{2}$ such that $(24/\pi_H - 2)\ \pi_H = 4\sqrt{2}_R\pi_H$. Then $\sqrt{2}_R = 1\frac{91}{222} = 1.4\overline{099}$. It is reasonable to assume that Rabbi Yohanan used $\sqrt{2}_Y := 1\frac{2}{5} + \frac{1}{100} = 1.41$ for $\sqrt{2}$.[33] The inaccuracy is $4\sqrt{2}_Y\pi_H - (24/\pi_H - 2)\pi_H = \frac{3}{2650} = 0.001132\ldots$ cubits.

Surprisingly, good approximations can be reconstructed *without* the assumption that Rabbi Yohanan knew the value π_H: for example, the *global* minimum of the weighted-error function

$$\sqrt{\left(\frac{\pi - x}{\pi}\right)^2 + \left(\frac{\sqrt{2} - y}{\sqrt{2}}\right)^2}$$

under the condition $(24/x - 2)x = 4yx$ is attained at $(x_0, y_0) = (3.136966\ldots, 1.412675\ldots)$. This gives independent mathematical evidence that more exact values were indeed used by Rabbi Yohanan.

3. SUMMARY

In summary, the following are the major approaches to the understanding of the Biblical and Talmudic value for π:

1. The rational-religious approach of Maimonides holds that, since we cannot know the exact values, the Bible tells us that we do not have to worry about this and that it suffices to use the value 3.[34]
2. The mystical approach of Munk contends that 3 was indeed the ratio of the circumference to the diameter in King Solomon's temple. This value is used in order to bridge the gap between our world and the "world of truth." For the sake of consistency, the halachic conditions are applied to the suitable regular polygons.
3. The practical approach of Rabbi Shimon Ben Tsemah asserts that the rough approximations are used when teaching the students, but, when it comes to practice, the calculations are to be done by the experts.

[32] "Note that $3\frac{15}{106}$, which is a lower bound, is a very interesting value, and may have been worked out also by Archimedes, although the evidence is ambiguous; Ptolemy's value is $3\frac{17}{120}$, which is a corresponding upper bound. It seems that these, or better values, were already known in the second century B.C.E., by Apollonius" [17]. See also [16, 157–158]. Note that π_H is the third convergent in the continued fraction of π [1, 96].

[33] The fraction $\frac{1}{100}$ occurs many times in the Rabbinical literature, mostly as is, and sometimes as $\frac{1}{10}$ of $\frac{1}{10}$. See Mishna Demai V 1; Maaser Sheni IV 8; Baba Kama VII 5. An even more precise approximation for $\sqrt{2}$ is given indirectly in Mishna Eruvin V 3. It is said there that twice the side of a square whose area is 5000 square cubits is equal to $141\frac{1}{3}$ cubits, i.e., $2\sqrt{5000} = 141\frac{1}{3}$, whence $\sqrt{2} = 1\frac{31}{75} = 1.41 + \frac{1}{300} = 1.41\overline{3}$.

[34] In his halachic sentences, Maimonides elegantly bypasses the irrationality problem by saying that the circle should be large enough to contain the square in question.

ACKNOWLEDGMENTS

The authors thank the following people for their help at various stages of this paper: Shlomo Edward Belaga of Louis Pasteur University; Wilbur Richard Knorr[35] of Stanford University; William C. Waterhouse of Pennsylvania State University; and Hussam Arisha Hag Ihia, Eliyahu Beller, and Jonathan Stavi—all of Bar-Ilan University. We owe special thanks to Victor J. Katz from the University of the District of Columbia for his helpful remarks. Finally, we thank the referees and editors for their part in bringing the paper to its current form.

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